Newton's Mountain Dome

Teaching How Bodies Orbit

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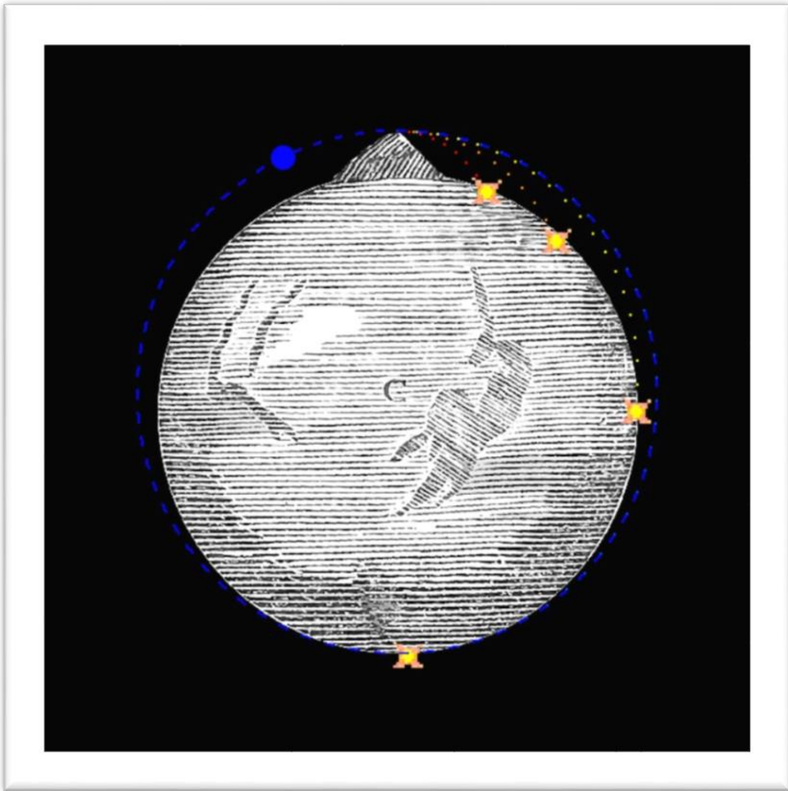

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This presentation offers details on how to use the "**Newton's Mountain Dome**" program to teach your students the basics of orbital motion.

Introduction

I have always found orbits to be a mostly mystifying subject for my students. I usually begin the discussion with the question "What causes bodies to orbit other bodies?" They universally answer "Gravity." I then posit that gravity is always an attractive force (as opposed to the electrostatic force which repulses as well as attracts) and therefore I ask "Then why don't bodies fall into the Sun?" This is where the fun begins. The myriad of answers that students suggest are fascinating (to put it politely and to quote Mr. Spock). Examples of their answers range from magnetism, other planets' pull on them (so how do you explain Pluto?), centrifugal forces (which don't exist in reality, i.e., centrifugal forces are fictitious, only appearing in a rotating reference frame), etc. Very rarely do I receive the answer that I'm looking for, that all orbits are actually a falling of one body around another. In other words, gravity ***always*** causes bodies to fall, but we typically don't recognize a body's falling trajectory as an orbit.

Newton addressed this issue in his short explanatory book *The System of the Worlds: Observing the Heavens*. In this book he presents a sketch of the Earth and various orbits created by a cannon on an impossibly high mountain as shown in Figure 1.

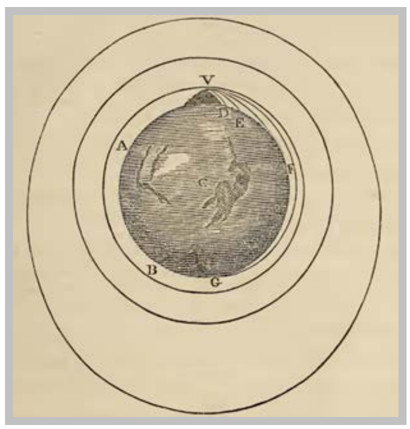

Figure 1 – The original sketch of Earth from Newton's *The System of the World*

What we have recreated in this program is an animated version of Newton's diagram which allows the user to shoot a cannonball at various horizontal and/or vertical velocities and then to trace out the orbits that these initial velocities create. If used properly the idea of orbits can be seen as simply a body ***falling*** around the Earth. Another feature of the program is the ability to "turn off" the Earth's surface so that the cannonball can travel within the Earth. That way the true nature of all trajectories can be seen as partial orbits where the Earth's surface simply gets in the way before the cannonball can return to its origin.

My experience in using this program to teach orbits has been ***extremely*** positive, typically giving my students their first <u>true</u> grasp of what orbits actually are. Leave it to Newton to best illustrate that all orbits are really one body falling around another. As Paul Valery said, "*One had to be a Newton to notice that the Moon is falling, when everyone sees that it doesn't fall*." - *Analects* (1966)

Teaching with the Program

You might begin be explaining that this demonstration is based upon Newton's own image published in his explanatory booklet *The System of the World* which he published because so few people could read and understand his masterpiece *Principia*. I often emphasize that if Newton felt that this was the best way to illustrate what an orbit truly is, then modernizing it by animating it is probably a good thing!

Begin by explaining that we've placed a powerful cannon on top of an impossibly high mountain (probably 800 miles high on this scale!) in order to experiment with the trajectories of cannonballs shot horizontally at differing speeds. We will begin with the default value of 0.5. Press the **Fire!** button and watch what happens. The cannonball will fly a short distance (giving off a loud bang – watch your volume!) and crash onto the surface as shown in Figure 2.

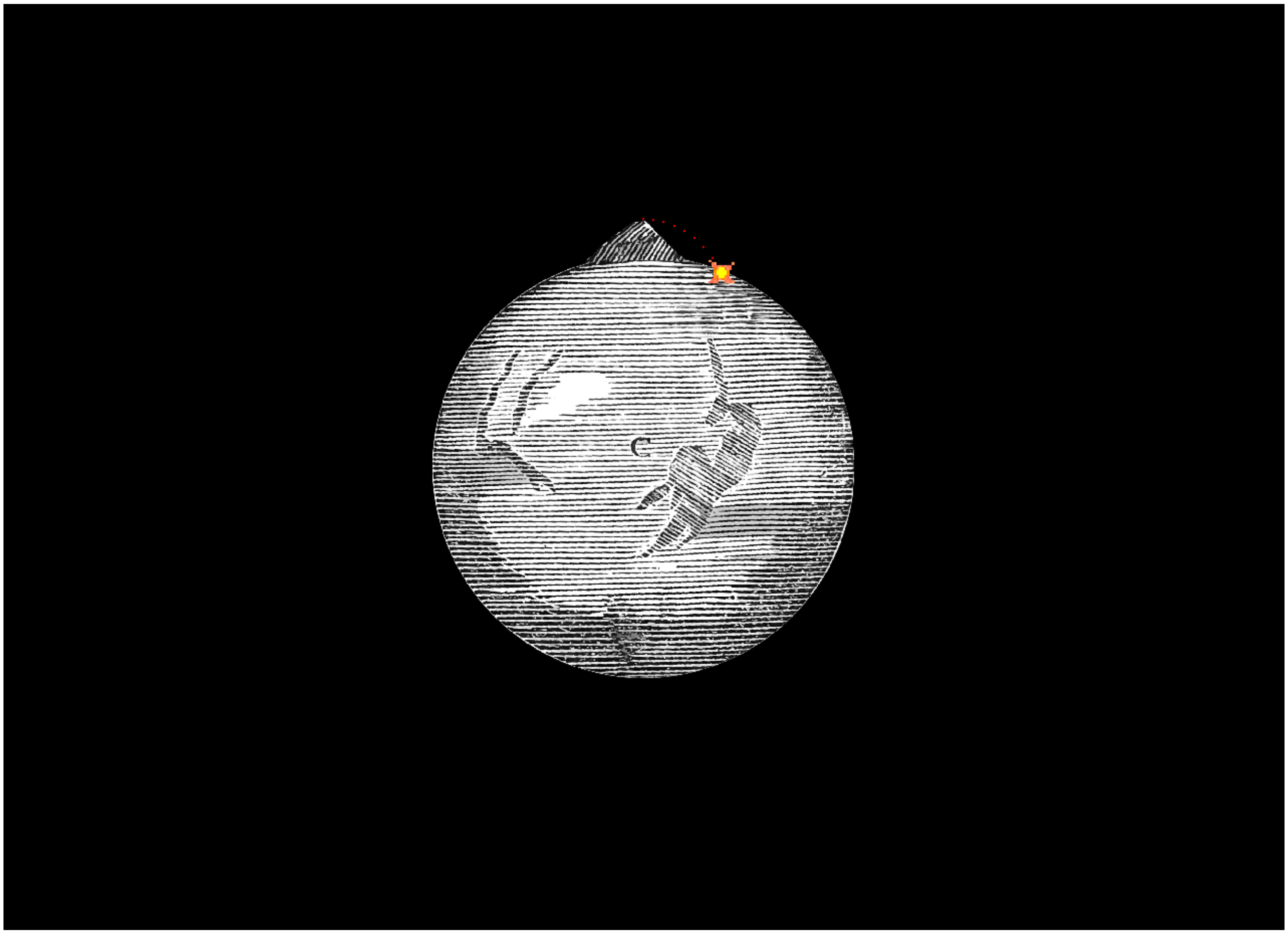

Figure 2 – Trajectory resulting from $V_x = 0.50$

Ask your audience "Why did the cannonball fall to the ground?" [The Earth's gravity is pulling it down towards its center.]

Again, ask "What can we do to increase the distance that it flies before it hits the surface?" [Increase the horizontal velocity.]

"Why would increasing the horizontal velocity cause it to fly farther before hitting the ground?" [Because the force of gravity can be considered as roughly constant near the Earth's surface, and therefore since its acceleration downward will be roughly constant, it will fall at the same rate no matter what the horizontal velocity is. Therefore it will travel farther horizontally since it will be pulled towards the surface at the same rate regardless of its horizontal velocity.]

Increase the horizontal velocity to a value of something like 0.70. Press **Fire!** again and you will see the trajectory as shown in Figure 3.

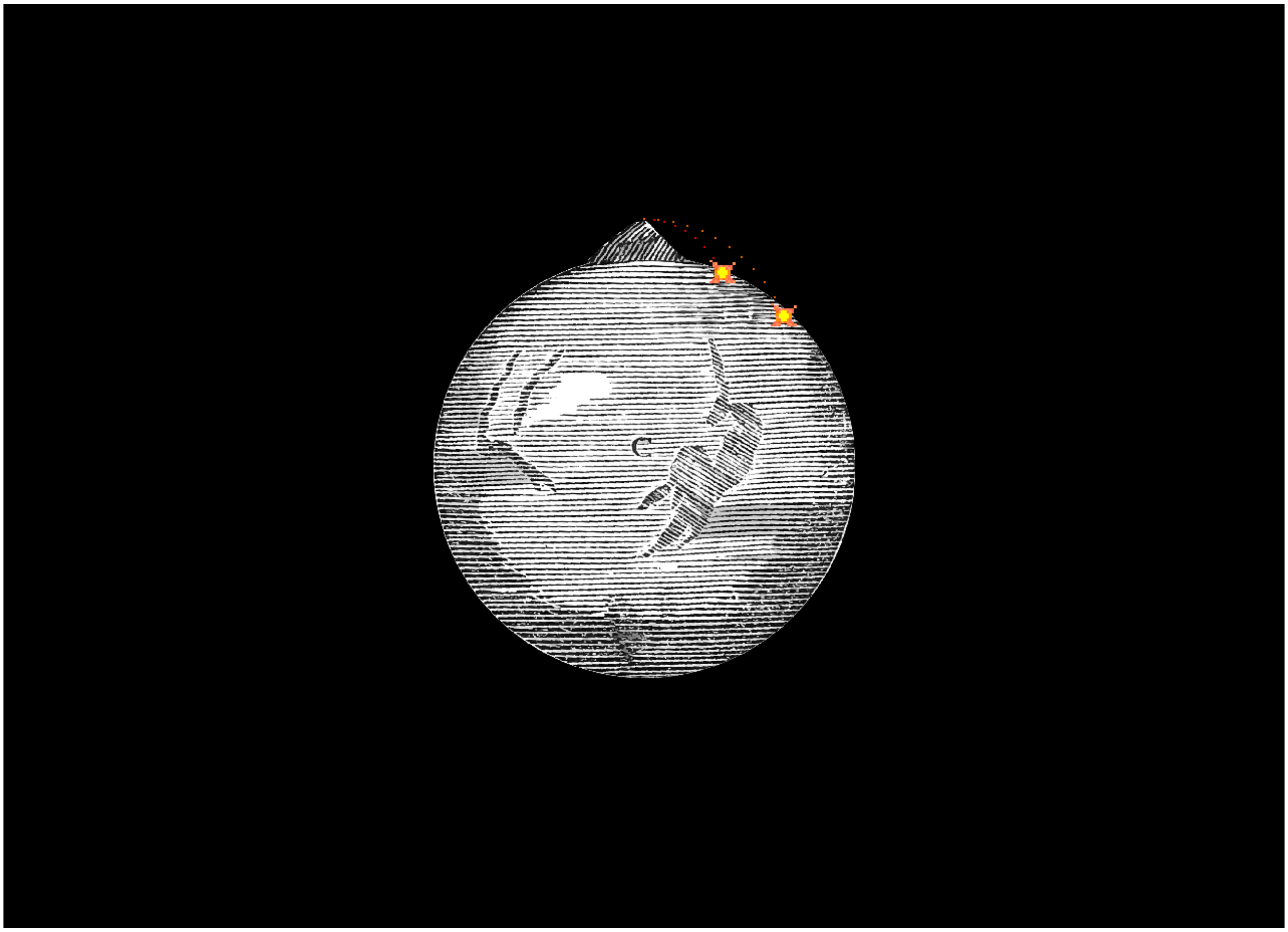

Figure 3 – Trajectory resulting from V_x = 0.70

Again, if we want to travel further, let's increase the horizontal velocity, this time to 0.85. Enter this value and press the **Fire!** button and you will see the trajectory shown in Figure 4.

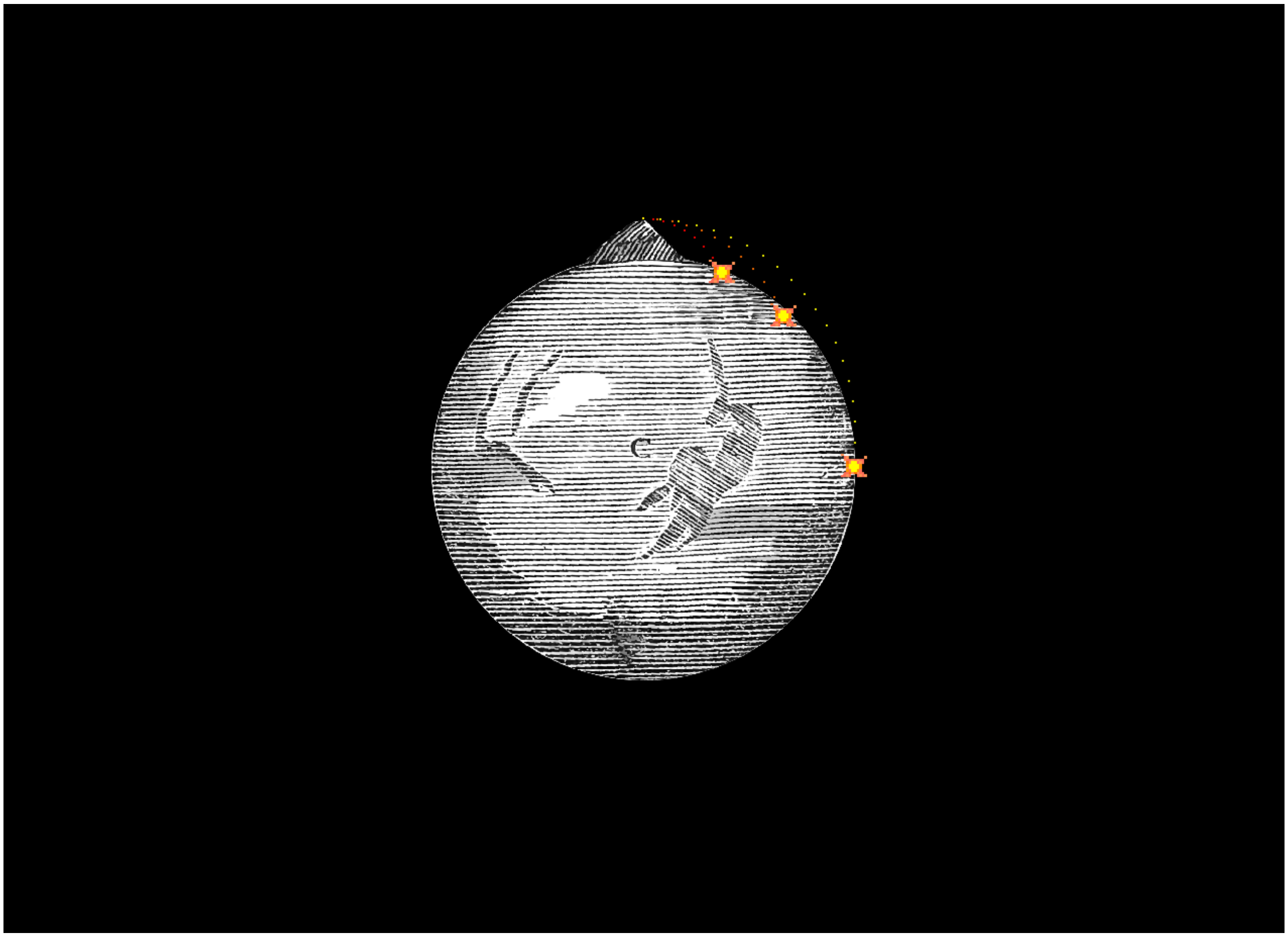

Figure 4 – Trajectory resulting from $V_x = 0.85$

Emphasize that each of these trajectories is occurring because the Earth's gravity is causing the cannonballs to fall towards its center, but its initial horizontal velocity is moving it a certain distance before the gravitational (vertical) acceleration is causing the cannonball to hit the ground.

Now, let's try an interesting number. Enter 0.8870 and then press the **Fire!** button. The resulting trajectory is shown in Figure 5. (You can of course try intermediate numbers leading up to this value, but that's up to you and how much time you have!)

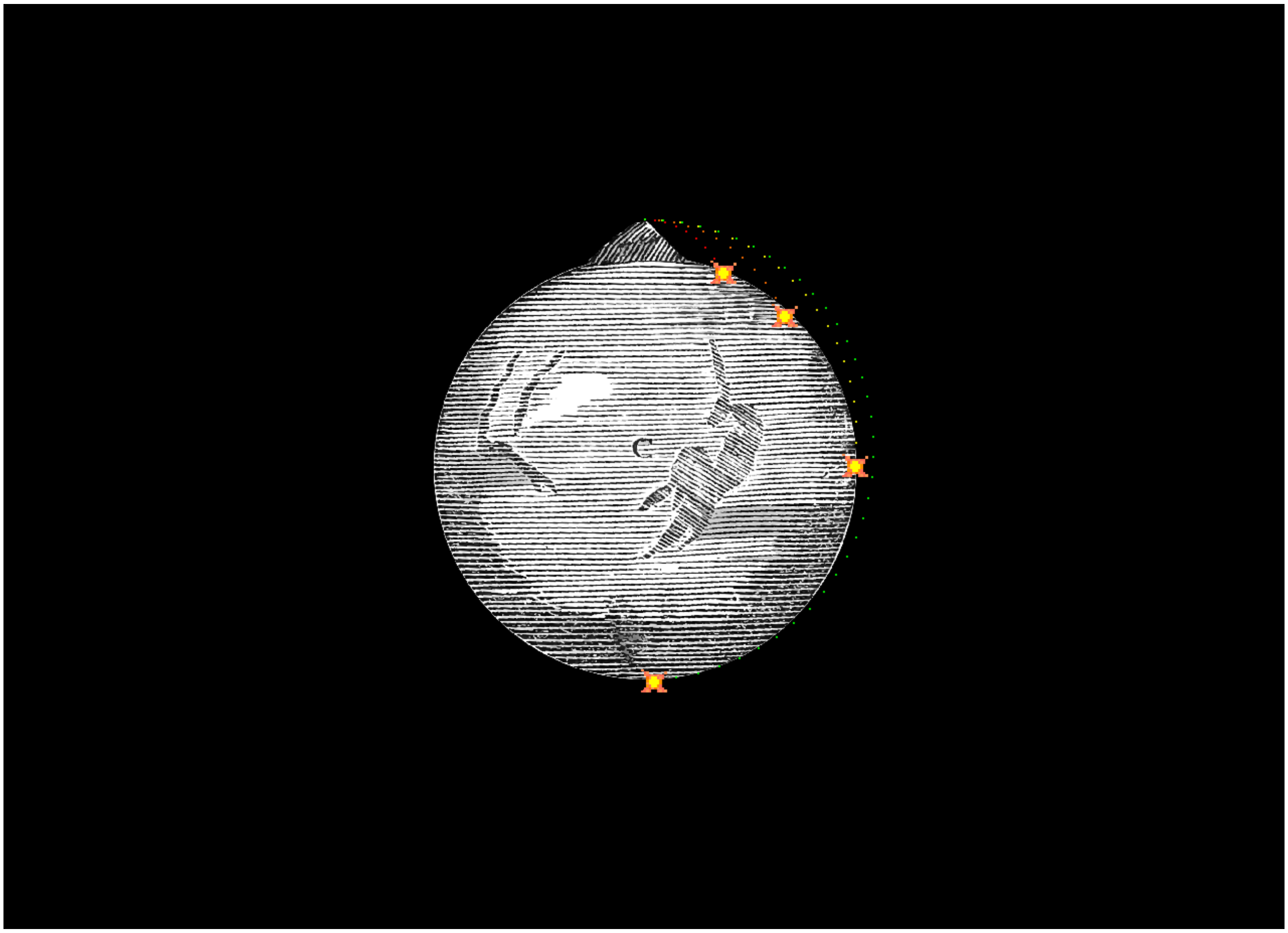

Figure 5 – Trajectory resulting from V_x = 0.8870

This interesting trajectory has resulted in a collision nearly exactly at the "bottom" of the Earth relative to the mountain's launching point. Hopefully the audience is beginning to see what's happening, but don't be surprised if your next shot throws them for a loop (literally).

Enter 0.8871 into the horizontal velocity and press the Fire! button. You will see a continuous orbit (Figure 6) and I guarantee that there will be gasps from some people in the audience (at least that's been my experience).

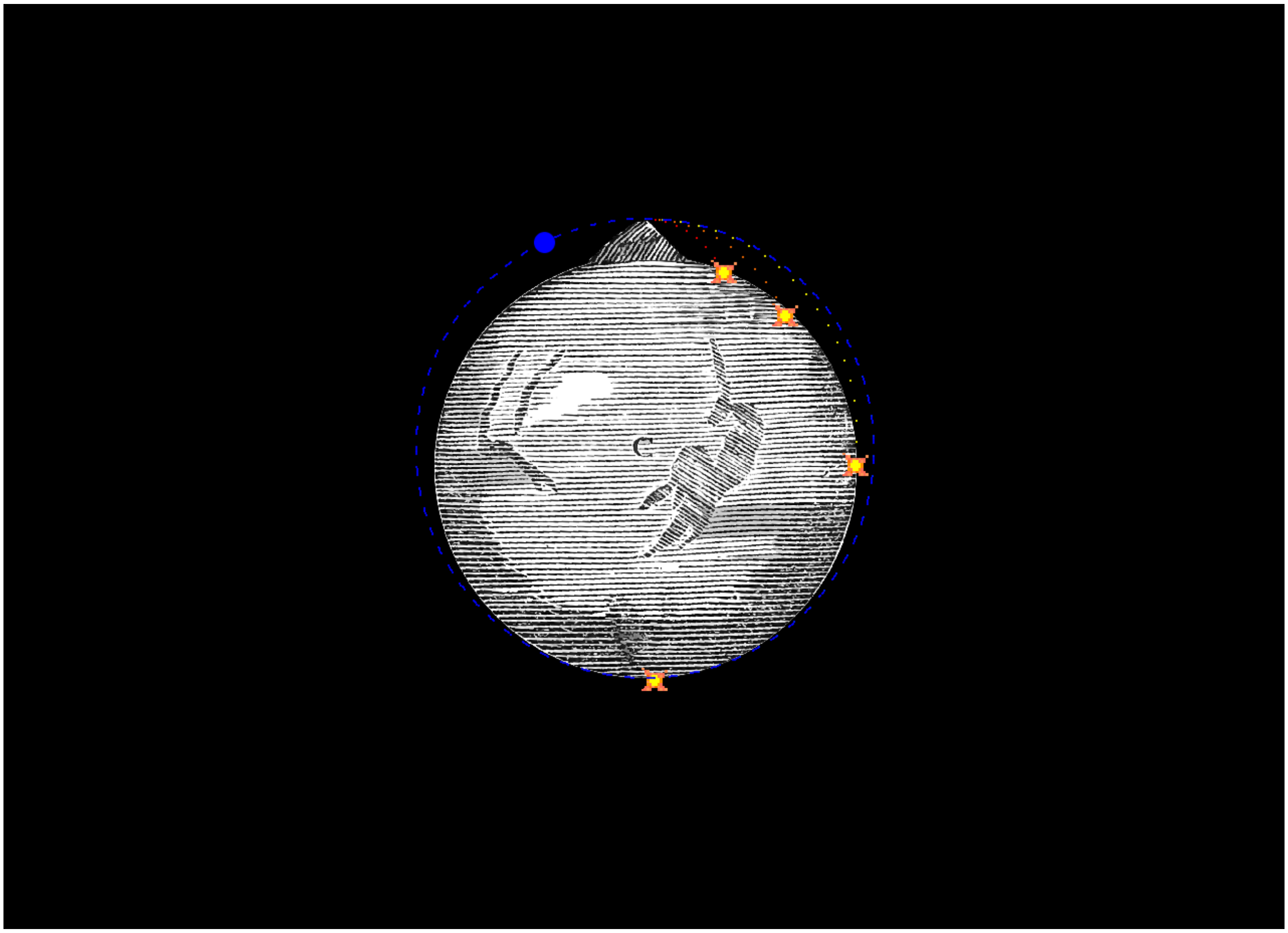

Figure 6 – Trajectory resulting from $V_x = 0.8871$ = continuous orbit!

Ask your audience, "What is happening? What is this cannonball doing? Why? How is it different from the other cannonballs that we fired?"

This last shot was no different from the others except that its initial horizontal velocity was high enough so that its rate of descent was the same as the rate of fall off of the curvature of the Earth! So, even though the cannonball is falling, it is falling continually and at a rate which insures that it will not strike the Earth's surface because the surface is "falling away" beneath it at the same rate at which it is falling towards the center of the Earth! This is exactly what an orbit is, a falling of bodies around each other. Gravity does indeed always cause objects to fall.

I usually stop here briefly and tell them that I can prove that orbiting bodies are in free fall around the Earth. My story goes something like this: When the astronauts traveled in the Space Shuttle, and they were apparently floating around inside the cabin, we say that they are in a state of ______________ (wait for the audience to fill in the blank) – weightlessness! But in reality, the force of gravity on the astronauts is only a few percent less in orbit than it is on the ground! (You can verify this by a simple calculation using Newton's law of gravity.) So what's happening? The Space Shuttle has a high enough horizontal velocity (at least 18,500 mph for a circular orbit) so that even though the Shuttle is falling under the influence of gravity, the rate at which it falls towards the Earth is the same as the rate at which the surface curves away from them. They aren't really weightless, but they feel that way because they (and the spacecraft containing them) are in free fall around the Earth.

If this doesn't make sense to your audience, then ask them to do this thought experiment: If you were in an elevator and the cable snapped and there were no safety devices to stop the free fall of the elevator, what would they be doing inside the elevator as it fell (besides screaming)? They would appear to be weightless because they would be in free fall. But obviously gravity is acting on them with full force (which they will soon experience when the elevator hits the bottom of the shaft!).

Next I emphasize that the previous trajectories that struck the Earth's surface were doing exactly the same thing as the last one, *i.e.*, attempting to orbit the Earth's center. This can be easily demonstrated by clicking on the checkbox labeled **Collisions** which will make the cannonball able to traverse the solid interior of the Earth with no resistance (see Figure 7). After clicking that checkbox, re-enter 0.50 into the horizontal velocity and then press **Fire!**. Your audience will watch the projectile orbit the center of the Earth, as shown in Figure 8.

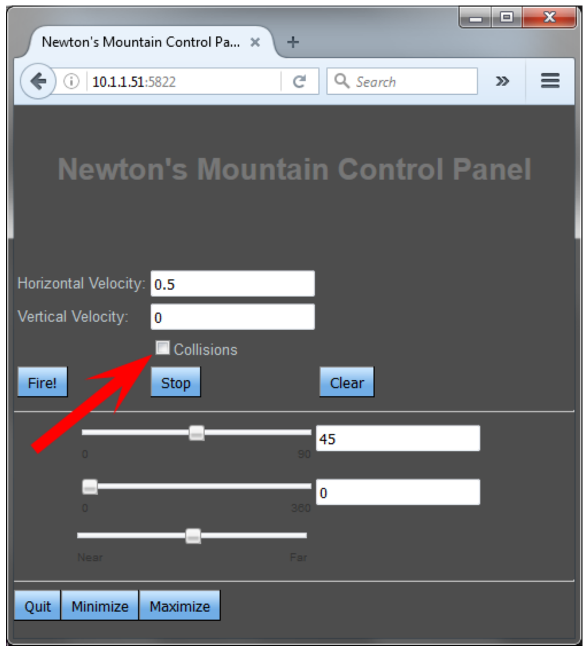

Figure 7 – Trajectory with V_x = 0.50 with Collisions turned off

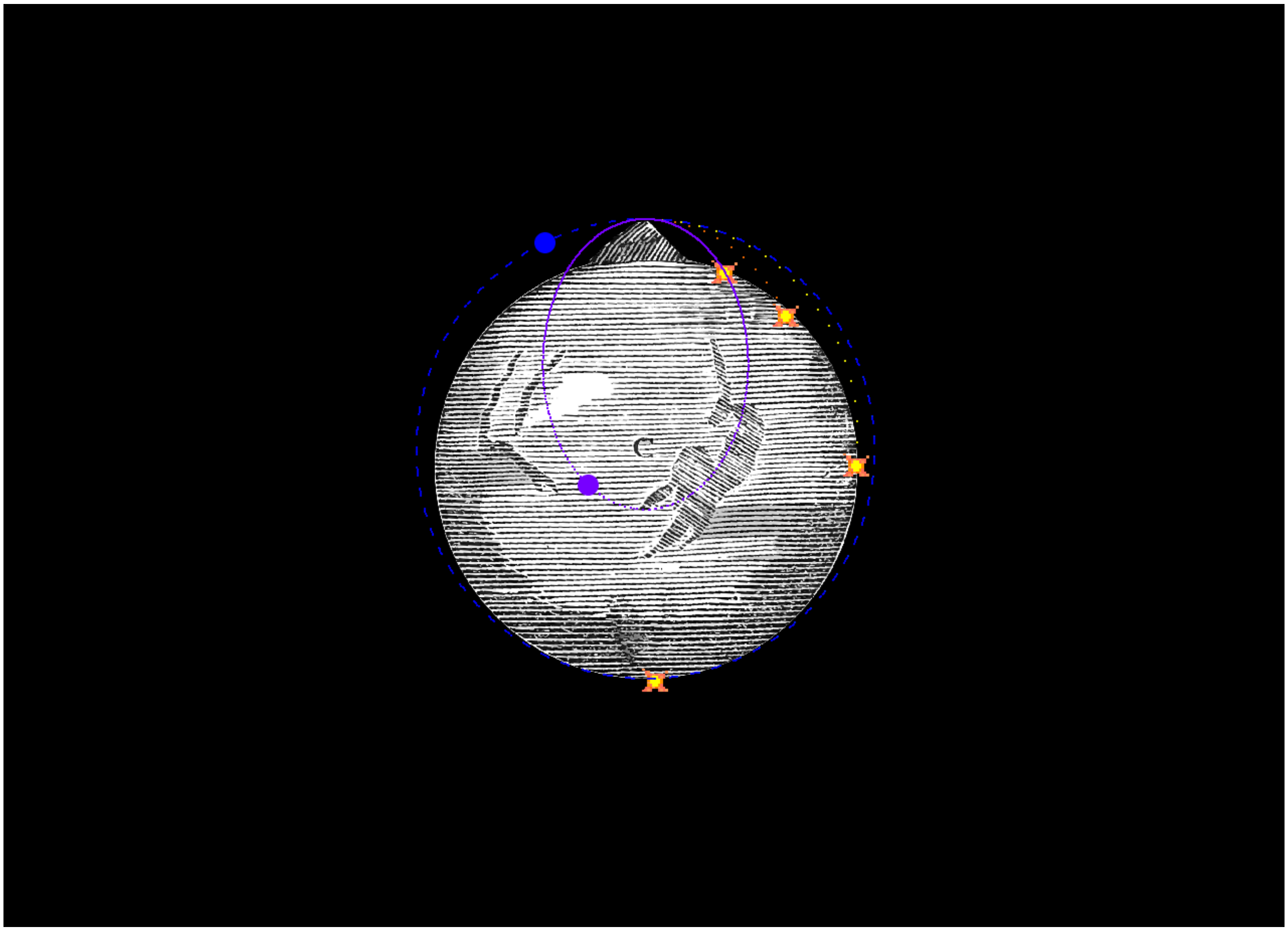

Figure 8 – Trajectory resulting from V_x = 0.50 with Collisions turned off

If you watch this orbit you will also see Kepler's Laws being demonstrated, which of course you can mention if you've already studied these together. I usually enter some even smaller horizontal velocity values to create more and more highly eccentric orbits. After showing these I emphasize that every time they throw a ball or kick a soccer ball, the ball is traveling in an orbit. Every projectile is in orbit, even if it's only for a very short while (until it hits the ground). The point of turning off the collisions is to emphasize the orbital nature of all trajectories above the Earth's surface. [If you *could* actually have a cannonball passing through the Earth's surface it would <u>not</u> behave as though all the mass is concentrated at its center (as in this simulation). Rather, the mass exterior to the traveling body inside the Earth is cancelled out by the mass on the other side of it. It only feels the gravity of the mass below it, not the mass above. At the center of the Earth there would be no force of gravity felt, being cancelled out by the symmetrical pull from all sides by the surrounding matter (assuming that it's symmetrically distributed)!]

Now, if time allows, you can have a little more fun entering ***larger*** numbers than the orbital velocity of 0.8871. Ask your audience what they would expect if you put in a horizontal velocity of 1.1? After they give some answers, enter this number and you will see the trajectory shown in Figure 9. (You will most likely have to increase the **Scale** value with the slider in order to see the entire orbit.)

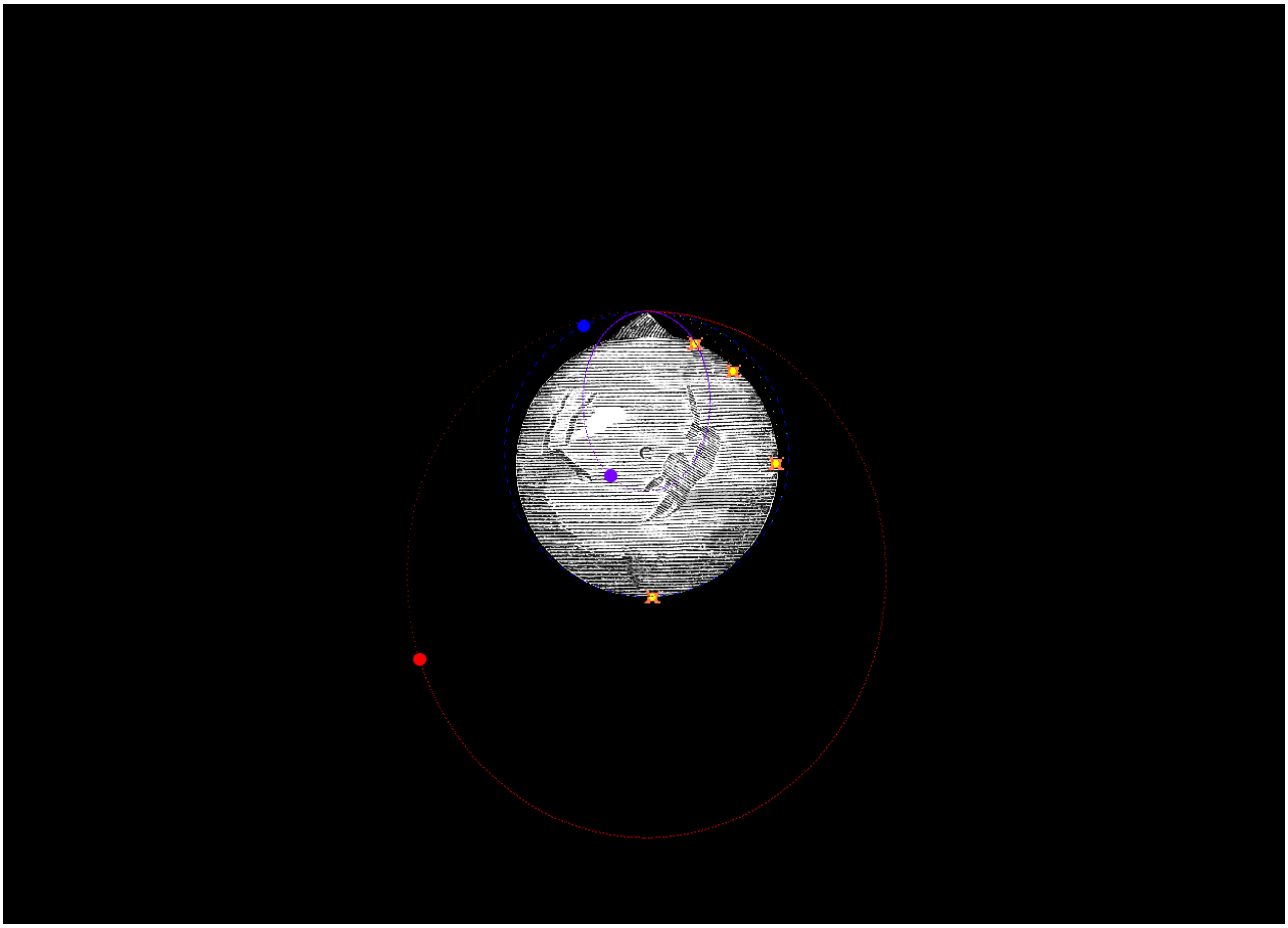

Figure 9 – Trajectory resulting from $V_x = 1.10$

Next try entering a number like 1.4. This will result in a hyperbolic orbit which will never return, *i.e.*, you have passed escape velocity. You can increase the scale to its max (see Figure 10) and show your audience that the projectile isn't going to return, as in Figure 11. Figure 11 also shows the trajectory resulting from an initial horizontal velocity of 10.0. (Usually I have to put in some ridiculously large number to keep the class happy!)

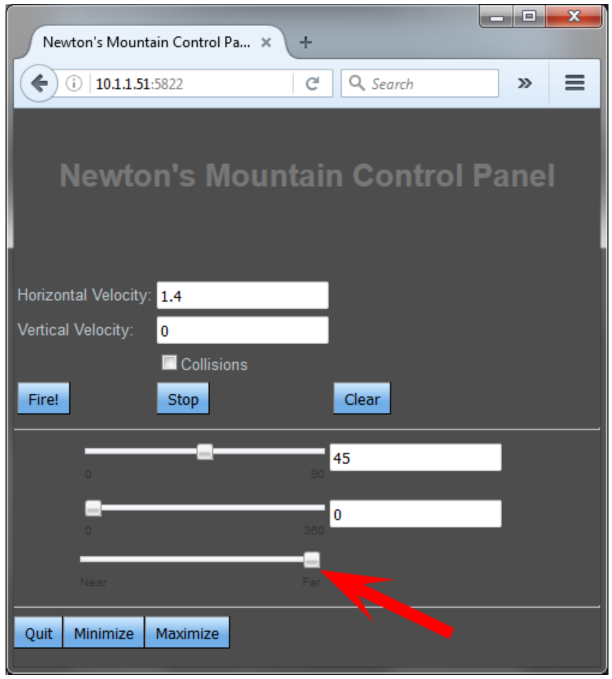

Figure 10 – Scale slider set to maximum scale for larger trajectories

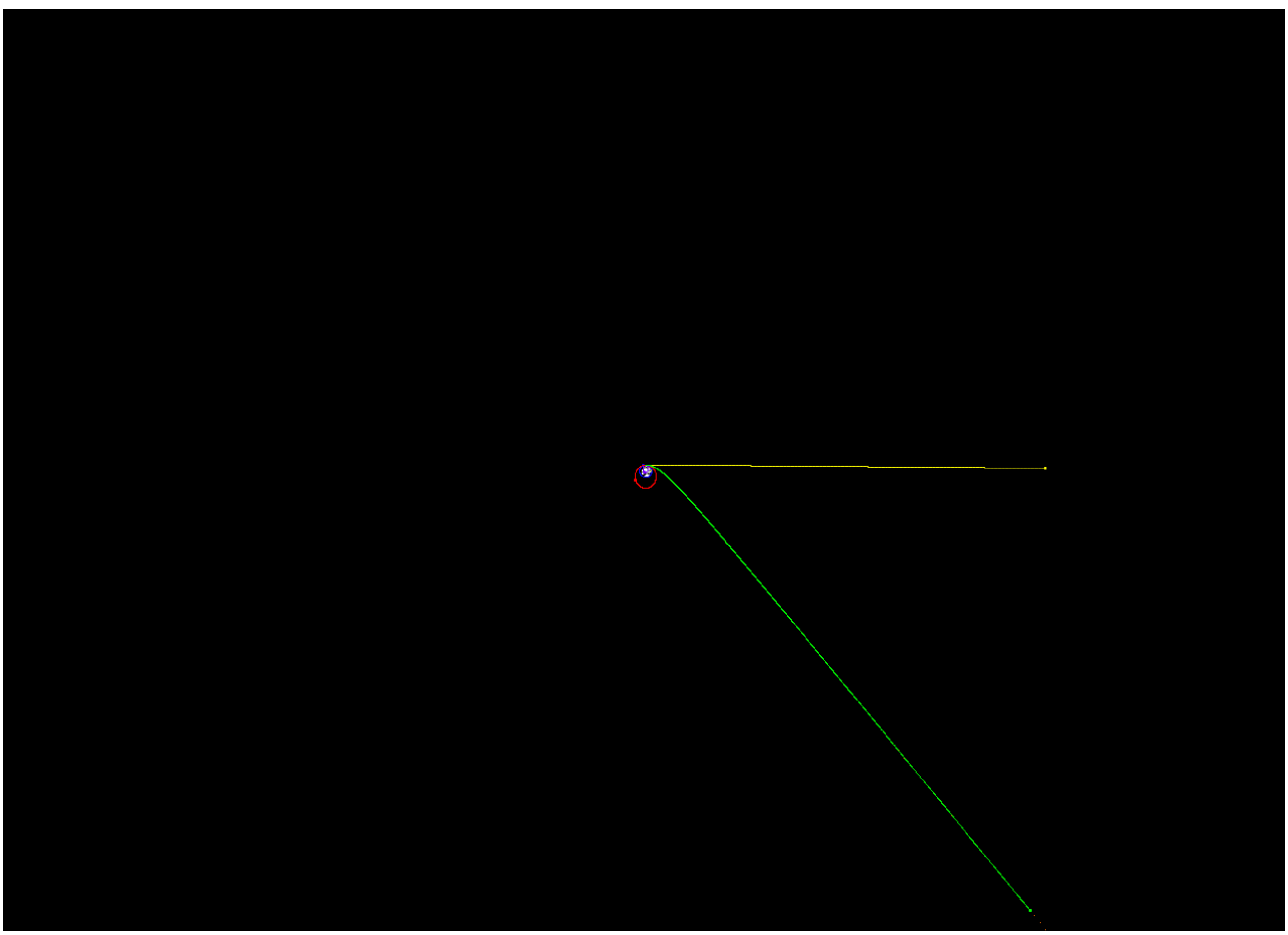

Figure 11 – Trajectories resulting from V_x = 1.40 (green) and V_x = 10.0 (yellow)

Don't be afraid to experiment with the program! Playing with vertical velocities can be lots of fun as well!

My main purpose in using this teaching tool is to demonstrate how orbits make sense with gravity always being a pulling force, *i.e.*, gravity always causes objects to fall (or be attracted to another object). My students have a *much* better grasp on these concepts after seeing this minilesson, and I suspect that it will become a mainstay in many of your presentations.