## Tides Dome

Teaching the Mystery of Tides and the Moon
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This presentation offers details on how to use the "Tides Dome" program to teach your students the basics of the role of differential gravitational forces from the Moon on Earth's tides, as well as the basics of vector addition.

## Introduction

The Moon's tides are one of the most confusing and difficult concepts to teach in astronomy. That the Moon causes the tides was known to the ancients, through the correlation of high tides and the position of the Moon in the sky. The mystery of why the Moon and not the Sun causes the tides had to wait until the $17^{\text {th }}$ century and the publication of Philosophice Naturalis Principia Mathematica by Isaac Newton. Despite tides being mathematically deciphered via physics for the past 4 centuries, few teachers today understand the cause and can successfully explain to students exactly why the Moon causes them. After all, most say it's the Moon's gravity that causes tides. But if that's the case, then the Sun should be the main cause of tides because its gravity is so much more powerful than the Moon's!

The actual answer is that it's the Moon's differential gravitational force and not simply its gravitational force. In other words, because of the Moon's proximity to Earth, it pulls about 6\% harder on the near side of the Earth compared to the far side. This means that the side closer to the Moon experiences a $6 \%$ greater gravitational acceleration than the far side. It is this differential gravitational force which causes the tides. This differential gravitational force depends on the size of the Earth compared to the distance between the Earth and Moon. The Sun's effect is so much smaller because the size of the Earth is so tiny compared to the distance between the Earth and the Sun. This distance is so great that the Sun essentially sees the Earth as a point mass and its gravitational force on the Earth is roughly uniform over the Earth's surface (but not zero - hence spring and neap tides).

The explanation above is okay...if you already understand tides! That's why I developed the Tides Dome program. What I will attempt in this User Manual is to explain how to load, run and explicitly how I use the program to teach tides to my undergraduate students. But it can be used at any level where you need to attempt to explain tides to any age.

## Teaching with the Program

The minilesson that I teach below is automatically "stepped through" using the bottom right hand button on the SciTouch. You can also step back steps by pressing the left bottom button on the SciTouch. For additional controls, please see the "Tides Controls" document.

I first emphasize that this diagram is not to scale, but that's okay for teaching purposes. Assuming that the program is running, display the vector at the center of the Earth which represents the Moon's gravitational force exerted on the center of the Earth. This vector is displayed by pressing the " 1 " key or simply clicking on the SciTouch's right bottom button (Mouse button $5=$ Lesson button) and the result is shown in Figure 1.


Figure 1 - Gravity vector displayed

Ask your students, "What will happen to the vector if I move the Moon in its orbit?" [The vector will swivel to always stay pointed at the Moon]. Figure 2 shows the gravity vector with the Moon located at 10 o'clock in its orbit. You move the Moon by holding down the left mouse button (or the top SciTouch button) and moving the mouse or SciTouch.


Figure 2 - Gravity vector displayed with Moon in different position
As obvious as this may seem, students enjoy watching the gravity vector swivel as you move the Moon in its orbit.

The force of gravity depends on the mass of the bodies involved and the distance separating them. To illustrate this, right mouse click (or trigger switch using the SciTouch) on the Moon and then move the Moon closer to the Earth. You will see the gravity vector increase in length as the square of the distance between the centers of the Earth and Moon. Likewise you can move the Moon away and the vector will decrease in length appropriately. See Figure 3.


Figure 3 - Gravity vector displayed with Moon at different distances from Earth
Now that we've shown how gravity depends upon the distance separating the two masses, we will show how it depends upon the masses of the bodies. Return the Moon close to its original distance from Earth. To double the mass of the Moon, press the " M " key or hold the lower right button (Lesson button) on the SciTouch and the Moon will appear to increase in size and its numerical mass will double. The gravity vector will double in length. Press the " N " key and the Moon's mass will return to its original mass. Press it once more (or hold the lower right button (Lesson button) on the SciTouch once) to halve its original mass which will make the gravity vector exactly half its original length. The resulting views of the two extremes are shown in Figure 4.


Figure 4 - Gravity vector displayed with Moon mass double (left) and halved (right)
Return the Moon to its original mass by pressing the "M" key (or Lesson button on SciTouch). We are now ready to begin investigating the Moon's gravitational forces on individual "pieces" of the Earth.

Ask your students, "How would the Moon's gravity affect the surface of the Earth closest to it, i.e., directly under it? Would it pull harder or less than it pulls the Earth's center?" [harder]
"What about the opposite side of the Earth directly opposite the Moon? Will it feel more or less force than the center of the Earth?" [less]
"What about the point at 12 'o clock on the Earth's surface? How would it feel the Moon’s pull differently than the center of the Earth?" [Almost the same magnitude because it's approximately the same distance away, but pointed in a slightly different direction since it's directed towards the Moon's center.]

Likewise, what about the point at 6 o'clock on the Earth's surface? How would it feel the Moon's pull differently than the center of the Earth?" [Almost the same magnitude because it's approximately the same distance away, but pointed in a slightly different direction since it's directed towards the Moon's center.]

Let's display these "cardinal" tide vectors and see if they make sense in light of the above discussion. Press the "2" key (or Lesson button on SciTouch) and you will see them displayed as shown in Figure 5.


Figure 5 - "Cardinal" gravity vectors displayed
I would recommend clicking on the Moon with the left mouse button (or hold in the Main (top) button on the SciTouch) and moving the Moon in its orbit to illustrate that indeed all of these vectors follow the Moon's center exactly.

Now let's discuss differential gravitational force. It should be obvious by now that the Moon pulls on the Earth differently depending upon how far away various parts of the Earth are from it. The greatest force acts on those parts closest to the Earth, the least force on those parts farthest away. We could also say that the gravitational acceleration on the near side of the Earth is greater than the gravitational acceleration on the far side. So the side closest to the Moon would accelerate more than the side farthest away.

One way of graphically depicting this difference in gravitational forces on the Earth by the Moon is by subtracting the gravity vector at the center of the Earth from each of the gravity vectors. This would show us the net force on the Earth as "felt" from the center of the Earth, i.e., a center of Earth force transformation. [I don't say that to my students. © ; Let's subtract the center gravity vector from each vector. To display this, press the "I" key (Lesson button on SciTouch) and the inverse of the gravity vector will be placed at the head of each vector in preparation for adding it to that vector. [Adding the negative is clearer than subtracting the actual vector - take my word for it!] You will see the following display (Figure 6) after zooming in by scrolling with the mouse wheel which zooms in and out, centered on the Earth. Remember that you can zoom in with SciTouch by placing the cursor on the Earth and using the rocker switch.


Figure 6 - The inverse central gravity vector added to each force vector
The vector solution to this operation is a vector from the base of the gravity vector (red) drawn to the head of the inverse gravity vector (orange). The most obvious case is that the center operation will result in zero (by definition of this transformation). The "near" operation (closest to the Moon) will result in a force vector which points to the right, starting at the base of the original force vector and ending at the head of the inverse vector, and so on. Press the "R" key (or Lesson button on SciTouch) to display the resultant vectors which represent the tidal forces that the Earth "feels" because of the nearby presence of the Moon as shown in Figure 7.


Figure 7 - The resultant vectors (white) representing the tidal forces created by adding the inverse central gravity vector to each force vector

Be certain that your audience understands what's going on here. There is no "white" resultant vector at the center since that result is exactly zero. The key here is that the resultant vector is always a vector which begins at the base of the original force vector (red) and ends at the head (arrow) of the inverse gravity vector (orange).

If you think your audience is with you, tell them that you are going to add more vectors at different places on the Earth's surface to better define the differential gravitational forces or tidal forces that the Earth experiences due to the Moon. Press the " 3 " key (or Lesson button on SciTouch) and these vectors will now be augmented by vectors placed at 30 degree intervals as shown in Figure 8.


Figure 8 - The resultant vectors (white) representing the tidal forces created by adding the inverse central gravity vector to each force vector at 30-degree intervals

Now let's double this coverage by placing force vectors at 15-degree intervals by pressing the " 4 " key (or Lesson button on SciTouch). The result is shown in Figure 9.


Figure 9 - The resultant vectors (white) representing the tidal forces created by adding the inverse central gravity vector to each force vector at 15-degree intervals

Let's attempt to clarify the situation. Since we're only interested now in the tidal forces (the small white vectors), let's hide all of the other intermediate vectors and display only the tidal force vectors. Press the "space bar" (or Lesson button on SciTouch) to display only the tidal force vectors as shown in Figure 10.


Figure 10 - The resultant vectors representing the tidal forces at 15-degree intervals
Let's further clarify this diagram by drawing a curve which passes through all of the heads of the tidal forces vectors. Press the "E" key (or Lesson button on SciTouch) to display this egg-shaped locus which illustrates the tidal force shape that the water on the Earth will attempt to mimic as it reacts to the tidal force of the Moon. This locus is shown in Figure 11.


Figure 11 - The tidal force surface
Assuming that the oceans attempt to take the shape of this egg-shaped surface, ask your students, "Where will high tide occur?" [At 3 and 9 o'clock, directly under the direction to the Moon]
"Where will low tide occur?" [At 12 and 6 o'clock, perpendicular to high tide, or $\sim 6$ hours apart from high tide.]

Zoom out using the mouse wheel (or point SciTouch to Earth and zoom out using the rocker switch) until you can see the Moon in its orbit again. Left mouse click (or hold in the Main (top) button on the SciTouch) and drag the Moon around in its orbit to emphasize that the egg-shaped ellipsoid follows the position of the Moon as shown in Figure 12. This should not be a surprise to your students!


Figure 12 - The tidal force surface follows the movement of the Moon

Place the Moon back at the 3 o'clock position. Zoom back in with the mouse wheel (or point SciTouch to Earth and zoom in using the rocker switch) until the Earth fills the field of view. Ask your audience once again (for emphasis) where high and low tides would occur.

The next feature we'll explore here is the fact that the Earth rotates once every 24 hours and yet the Moon only moves $\sim 13$ degrees in this time. Therefore, to first approximation, the Earth is attempting to rotate "beneath" the water surface, and hence why we experience high to low tides every 6.5 hours or so. (It's not six hours because the Moon is revolving about 1 hour of angle each day and so the tidal ellipsoid moves about 1 hour or 13 degrees in the direction of the Earth's rotation. So it takes an extra half hour or so for the Earth's surface to "catch up.") To illustrate this concept, press the " S " key (or Lesson button on SciTouch) to watch the Earth rotate beneath its tidal force ellipsoid. This should help solidify why we have low and high tides approximately every 6.5 hours.

Finally, demonstrate the combination of the Earth's rotation and the Moon's revolution by pressing the "O" key (or Lesson button on SciTouch) and you will see the Moon revolving around the Earth in scale with the Earth's rotation rate.

Your students may also note that the high tide opposite the Moon's position would seem to be slightly less high than the one directly under it. This is indeed the case, although it typically only amounts to a foot or two difference in maximum height. I've never seen this indicated on any drawings in textbooks, but a simple calculation using Newton's Law of Gravity shows that this must be so, and of course if you consult any local high and low tide calendars which indicate water levels you will see that this is reality.

At this stage you can play with changing the Moon's mass and/or distance from the Earth and watch how it affects the tidal surface. You can emphasize that, the further away the Moon is, or the weaker is its gravitational force, the smaller will be the tidal forces. The opposite of this will also be true.

I have found this program to be of tremendous help in clarifying tides to my students as well as science educators! Have fun with it!

